JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 17, No. 2, March–April 1994

Multiobjective Trajectory Optimization by Goal Programming with Fuzzy Decisions

Shinji Suzuki* and Takeshi Yoshizawa†
University of Tokyo, Bunkyo-ku, Tokyo 113, Japan

A sequential goal programming approach is considered for not only well-defined flight trajectory problems but also ill-defined problems that have no feasible solutions satisfying all design requirements due to strict boundary conditions or tight path constraints. By using a time integration algorithm, trajectory optimization problems are transformed into numerical optimization problems that seek optimal control variables at discrete time points to minimize an objective function and satisfy various design constraints. By defining the target goal values of both the constraints and the objective functions and by prioritizing each goal according to its significance, the GP formulation modifies ill-defined problems as multiobjective design problems. Additionally, a fuzzy decision making method is applied for those goals that are prioritized, not precisely, but in a fuzzy manner. Numerical applications for simple ascent trajectory problems show that this method can efficiently find the trajectories when various kinds of design requirements are imposed for the ill-defined problem.

Introduction

A FLIGHT trajectory problem can be formulated as an optimal control problem that finds the solution that maximizes or minimizes an objective function and satisfies both boundary conditions and path constraints.[1] Although numerous solution approaches have been developed for mathematically well-defined problems, real-world flight trajectory design problems are not always formulated as well-defined mathematical problems. Ill-defined problems refer to the following situation: constraints on control and state variables are so severe that no permissible flight path satisfying all boundary conditions can be found. In this situation, some flight trajectories may be required even if some of the boundary conditions or the constraints must be relaxed to accommodate other objectives using various decision making methods. The constraints or conditions that cannot be achieved should be regarded as goals. Consequently, multiple goals or objectives, rather than a single objective, must be considered. This paper describes flight trajectory problems with multiobjectives that are solved by a sequential goal programming approach combined with fuzzy decision making.

A number of authors have applied a parameter optimization method for optimal trajectory problems that have a single objective. Hargraves and Paris[2] presented the method that employs polynomial splines to represent both control and state variables and uses collocation to transform differential equations into algebraic equations. They solved parameter optimization problems by sequential quadratic programming, showing the efficiency and the robustness of the method. Betts and Huffman[3] have further developed this method by introducing sparse matrix methods to solve large problems such as an aerodynamic re-entry problem with high nonlinearity. The basic ideas of the direct numerical solution algorithms for optimal trajectory problems were earlier investigated by Johnson,[4] Williamson,[5] Rader and Hull,[6] and others. The direct approach has been recognized to apply to very general trajectory problems and has been proposed for real-time navigation.[7]

In the present paper, the time scale is divided into a set of elements so as to utilize a modified Euler integration method for trajectory dynamics. Accordingly, control variables are represented as discrete values at each nodal point. For the first time, a sequential goal programming algorithm is employed for trajectory optimization problems having multiple objectives. Goal programming (GP) was developed to apply linear programming to multiple criteria optimization problems by Charnes and Cooper,[8] Ijiri,[9] Lee,[10] and Ignizio.[11] GP introduces target values (goal values) of achievement for each objective or constraint and specifies classes of significance for each of the goals. In a flight trajectory design problem, it may be possible to define significant classes for the following goals: boundary conditions, control and state variable constraints, and design objectives. The GP method is advantageous because it can successfully find a trajectory that achieves goals according to their priority classes, even if all of the goals cannot be satisfied. If the problems are linear with respect to design variables, GP problems can be solved efficiently by a conventional simplex algorithm. To utilize these advantages, a sequential linearization method has been proposed for nonlinear GP problems to solve simultaneous structure-control optimization problems for a wing structure with an active control system.[12,13]

Cases exist in which goals cannot be prioritized in a strict sense, but the achievement level of multiple competing goals may be determined by the decision maker's vague criteria. We employ a fuzzy decision making method proposed by Bellman and Zadeh[14] to deal with the decision maker's vague ideas or the fuzzy characteristics of problems by using fuzzy set theories introduced by Zadeh.[15] Zimmermann[16,17] employed the fuzzy decision making method in linear programming problems. To address a real-world design process, the fuzzy optimization method has been applied to structure optimization problems,[18] to the design of mechanical and structural systems,[19] and to the flight profile design and mechanical design of a helicopter.[20]

An optimal ascent trajectory problem with thrust direction control is studied for numerical examples. First, single-objective problems that have feasible solutions are solved to check the accuracy of the sequential GP approach presented in this

Table 1 Weight selections in goal programming

Weight	Constraint or objective	
$w_j^+ > 0,\ w_j^- > 0$	subject to	$f_j(p) = g_j$
$w_j^+ > 0,\ w_j^- = 0$	subject to	$f_j(p) \le g_j$
$w_j^+ = 0,\ w_j^- > 0$	subject to	$f_j(p) \ge g_j$
$w_j^+ > 0,\ w_j^- < 0$	minimize	$f_j(p)$

Received July 10, 1992; revision received June 7, 1993; accepted for publication June 7, 1993.

*Associate Professor, Department of Aeronautics, 3-7-1. Member AIAA.

†Graduate Student, Department of Aeronautics.

paper. Secondly, we investigate ill-defined problems that have no feasible solutions due to unachievable boundary conditions or constraints. Finally, a fuzzy optimization approach combined with the sequential GP formulation is proposed to solve multiobjective trajectory optimization problems.

Trajectory Optimization Problems

Mathematical Formulation

A system is subjected to the state equations

$$\dot{x} = f(x, u, t) \tag{1}$$

where x and u are the vector of states and the vector of controls, respectively. Boundary conditions may be set as

$$\psi_I(x(t_I)) = 0, \qquad \psi_F(x(t_F)) = 0 \tag{2}$$

where t_I and t_F are the initial and final time, respectively.

During a flight path, constraints are defined as

$$L_{PC} \le S(x, u, t) \le U_{PC} \tag{3}$$

Whereas $S(x, u, t)$ contains control variables, our formulation allows pure state constraints, i.e., $S(x, u, t)$ is replaced by $S(x, t)$ in Eq. (3).

The control function $u(t)$ is sought to optimize a given performance index

$$J = \phi[x, u, t] \tag{4}$$

where x and u must satisfy state equations [Eq. (1)], boundary conditions [Eq. (2)], and path constraints [Eq. (3)]. Although numerical examples that appear in this paper consider a performance index defined at the final time, a performance index can be given as a functional of time.

Nonlinear Programming Formulation

The optimal control problem stated in the preceding section is converted to a nonlinear programming problem. To transform the control function $u(t)$ to a set of discrete variables, the time interval from t_I to t_F is divided into N elements, being the nodal values of the time denoted by t_i [$i = 1, 2, \ldots, (N+1)$]. Therefore, control variable $u(t)$ is represented as a set of discrete values $u_i = u(t_i)$.

If the initial states $x(t_1)$ are specified in the initial boundary conditions, and if the control variables u_i and the final time t_F ($= t_{N+1}$) are assumed, the state variables $x(t)$ can be calculated using an appropriate integration scheme. The modified Euler method (see the Appendix) for numerical examples is employed because it is efficient. Consequently, the nodal values of the states are written as functions with respect to both u_i [$i = 1, 2, \ldots, (N+1)$] and t_F. These independent variables are combined into a single vector p as

$$p^T = [u_1^T, u_2^T, \ldots, u_{(N+1)}^T, t_F] \tag{5}$$

Note that where some of the initial states are unknown, they can be added to the independent variables p.

The performance index J, the boundary conditions at t_I and t_F, and the path constraints are functions with respect to the independent variables p. Therefore, the trajectory optimization problem can be formulated in the following nonlinear programming problem:

$$\begin{aligned} &\text{minimize} && \phi(p) \\ &\text{subject to} && \psi_I(p) = 0 \\ &&& \psi_F(p) = 0 \\ &&& L_i \le S_i(p) \le U_i \\ &&& i = 1, 2, \ldots, (N+1) \end{aligned} \tag{6}$$

where the path constraints are imposed at each nodal point t_i.

The problem formulated in Eq. (6) can be written in the following standard nonlinear programming problem:

$$\begin{aligned} &\text{minimize} && \phi(p) \\ &\text{subject to} && c_j(p)\{\le, =, \ge\}b_j \\ &&& j = 1, 2, \ldots, n_p \end{aligned} \tag{7}$$

where $c_j(p)$ are n_p constraint functions.

Sequential Goal Programming Formulation

A nonlinear programming formulation generally requires an initial solution. In a flight trajectory problem, it is quite difficult to find solutions satisfying all constraints since the boundary conditions are imposed not only at the initial time but also at the final time. Some methods have been utilized to overcome this difficulty. A penalty function approach introduces some penalty to limit constraint violations and adds this penalty function to the original objective functions. Although this approach is simple, the penalty function often leads to problems of numerical calculations. Furthermore, it is difficult to determine the value of a weight introduced in the combined objective function. The gradient-restoration method, presented by Miele et al.,[21] is a different approach. They divided the original problem into two and solved each alternately: first, the minimization of an objective function and second, the satisfaction of constraints. This paper utilizes a GP approach,[8-11] which is suitable for this kind of optimization problem. GP deals with both an objective function and constraints as goals to be satisfied and introduces the concept of the "priority level,"[10] which represents the importance of each goal. Since the priority level means the "preemptive" priority, goals in the high priority level must be satisfied before we consider goals in the low level. GP is able to solve optimal flight trajectory problems with a simple formulation by regarding constraints in the high priority level and an objective function in the low priority level. Furthermore, GP can accommodate the situation in which no feasible solutions are found that satisfy all constraints. Although the penalty function approach can solve this ill-defined problem, choosing priority levels for unsatisfied constraints is easier than defining penalty functions.

In a GP formulation, goal values are defined for the constraint functions that are not achieved; hence, deviation vari-

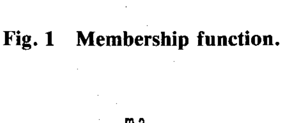


Fig. 1 Membership function.

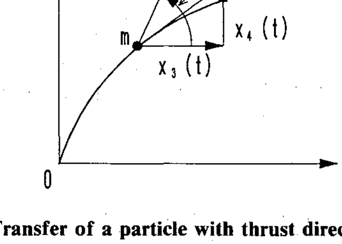


Fig. 2 Transfer of a particle with thrust direction control.

ables d^+ and d^- are introduced to measure the overachievement and the underachievement of the target goal. If the performance index $\phi(p)$ is represented as $f_1(p)$ and has the target goal value g_1, and if the constraint functions denoted by $f_2, \ldots, f_{(n_g+1)}$ (n_g is the number of unachieved constraints) have target values $g_2, \ldots, g_{(n_g+1)}$, both the objective function and the constraint functions can be dealt with equally. Thus, we have

$$\text{subject to} \quad \begin{aligned} & f_j(p) - d_j^+ + d_j^- = g_j \\ & d_j^+ \cdot d_j^- = 0 \\ & d_j^+ \geq 0, \; d_j^- \geq 0 \\ & j = 1, 2, \ldots, (n_g + 1) \\ & c_k(p)\{\leq, =, \geq\} b_k \\ & k = 1, 2, \ldots, n_p - n_g \end{aligned} \tag{8}$$

By using the deviation variables, the objective functions to be minimized are defined as

$$w_j^+ d_j^+ + w_j^- d_j^-, \qquad j = 1, 2, \ldots, (n_g + 1) \tag{9}$$

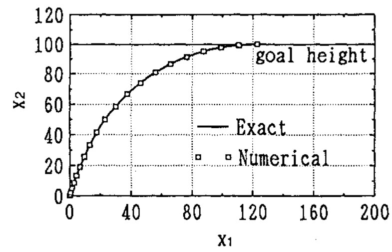
where w_j^+ and w_j^- are weights that are appropriately chosen to deal with various types of design requirements, e.g., inequality constraints, equality constraints, and function minimization as shown in Table 1. Table 1 indicates a way of selecting the weights in Eq. (8). For example, when the exact achievement of the goals is desired (this situation is called equality constraints), both w_j^+ and w_j^- must be represented as positive. When underachievement of the goals is acceptable (it is called inequality constraints), negative deviations d_j^- should not be included in Eq. (8); thus, w_j^- must be zero. And when the minimization of the functions f_j is desirable, w_j^+ and w_j^- must be positive and negative, respectively.

Each goal has the priority of significance in an engineering sense, and the goals are grouped according to their priority. Priority levels P_l ($l = 1, 2, \ldots, L$) are then introduced to make the following objective function

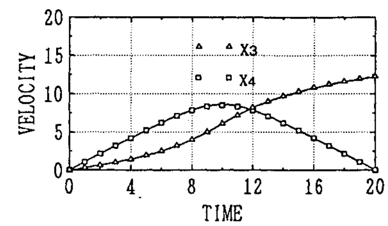
$$\text{minimize} \sum_{l=1}^{L} \left\{ P_l \left[\sum_{j \in I_l} (w_j^+ d_j^+ + w_j^- d_j^-) \right] \right\} \tag{10}$$

where I_l denotes a set of goals that are categorized in the same priority level l. P_l is called the preemptive priority factor.[10] This factor satisfies the relationship of $P_i >>> P_{i+1}$, which implies that the goals at the upper priority level should be considered to be infinitely more important than goals at the lower priority level. The decision making process used in Eq. (10) is stated in the following manner: the goals at the upper priority level must be achieved to the decision maker's satisfaction before we can consider the goals at the lower priority level. To assign weights to goals of the same priority level, we must select appropriate values of w_j^+ and w_j^- for each goal.

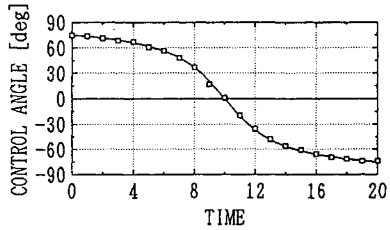
This kind of optimization problem is categorized as preemptive (lexicographic) goal programming, which can be solved efficiently by a modified simplex algorithm[9] if all of the functions in Eq. (8) are linear with respect to p. To utilize this modified simplex algorithm, a sequential GP formulation is applied in the following manner: 1) the initial values of each variable p are assumed; 2) all of the functions in Eq. (8) are linearized with respect to p; 3) the optimal solutions are obtained within the move limits[22] of each design variable, where move limits are defined as the upper and lower limits of each design variable; and 4) the variables p are successively updated until all of the solutions converge. Linearization with respect to each variable is implemented by the use of a finite difference method in the numerical examples in this paper. The move limits should be selected that assure the linearity assumption and converge the iteration process in a conventional sequential linear programming approach.[22] Initial values of the move limits that have large values to help reduce the number of iterations are gradually shortened when a parameter changes its sign during the iteration process.



a) **Flight trajectory x_1 vs x_2.**



b) **Time histories of velocity components x_3 and x_4 vs t.**



c) **Time history of control angle β vs t.**

Fig. 3 Solutions of problem A (well-defined).

Trajectory Optimization with Fuzzy Decisions

The GP formulation presented in this paper can solve flight trajectory problems by classifying the significance of goals. In a real-world flight trajectory design, distinct classification of the significance of each goal may not be specified but can appear in a fuzzy manner. A fuzzy decision making method can be applied to a flight trajectory design.

In a fuzzy environment, the decision maker can define the following membership function: it has the value of zero for the worst possible case, the value of one for the best possible case, and the intermediate value for those cases in between. Although any type of membership function can be employed, the present paper utilizes a linear interpolation function. Figure 1 illustrates a membership function used in this paper. It indicates the degree of the decision maker's satisfaction with the final time of a flight trajectory: the final time of 20 s is considered best and 25 s worst. At these points membership functions are one and zero. Although it is possible to change the shape of membership functions during the iteration process, in this paper the shape is fixed.

When maximization of goal functions is desired, membership functions for n_f goals are defined as

$$\mu_i(g_i) = \begin{cases} 1 & \text{for } g_i \geq g_i^* \\ 1 - \dfrac{g_i^* - g_i}{g_i^* - g_i'} & \text{for } g_i' \leq g_i \leq g_i^* \\ 0 & \text{for } g_i \leq g_i' \end{cases}$$

$$i = 1, 2, \ldots, n_f \tag{11}$$

where g_i is the value of the goal function $[g_i = f_i(p)]$, and g_i^* and g_i' are the maximum and minimum goal values, respectively.

The membership function of the solution-set is then[17]

$$\lambda_D = \min\{\mu_1, \mu_2, \ldots, \mu_{n_f}\} \tag{12}$$

and the "maximization decision"[17] yields the following min-max programming problem

$$\begin{aligned} &\text{maximize} && \lambda_D(p) \\ &\text{subject to} && c_j(p)\{\leq, =, \geq\}b_j \\ & && j = 1, 2, \ldots, n_{uf} \end{aligned} \tag{13}$$

where the constraints $c_j(p)\{\leq, =, \geq\}b_j$ represent the n_{uf} unfuzzy constraints. By introducing a new independent variable λ, Eqs. (13) can be transformed into

$$\begin{aligned} &\text{maximize} && \lambda \\ &\text{subject to} && \lambda \leq \mu_i \\ & && i = 1, 2, \ldots, n_f \\ & && c_j(p)\{\leq, =, \geq\}b_j \\ & && j = 1, 2, \ldots, n_{uf} \end{aligned} \tag{14}$$

The fuzzy approach for goals can also be formulated in a standard nonlinear programming problem that can be utilized for goals in the same priority class.

Numerical Examples

Well-Defined Trajectory Optimization

To demonstrate the validity of the method, two planar powered flight cases that have analytical solutions were

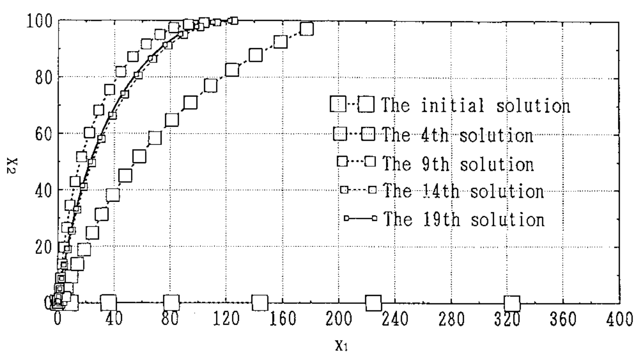


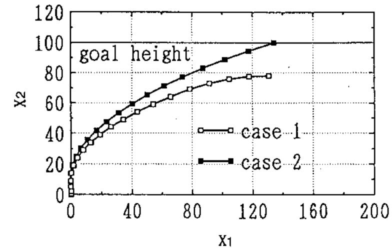
Fig. 4 Iteration history of the flight trajectory in problem A.

Table 2 Number of iterations and relative error for problems A and B

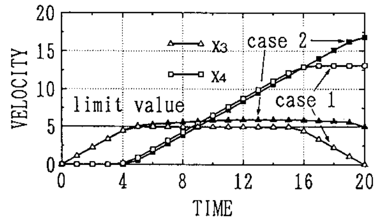
Problem	Number of elements	Number of iterations	Relative error
A	2	20	3.40E-1
	4	17	1.18E-1
	6	21	5.73E-2
	8	17	3.34E-2
	10	23	2.17E-2
	16	20	8.71E-3
	20	19	5.69E-3
	40	28	1.41E-3
B	2	24	5.17E-2
	4	26	1.78E-2
	6	26	8.40E-3
	8	22	4.79E-3
	10	31	3.10E-3
	16	33	1.22E-4
	20	24	8.03E-4
	40	28	2.13E-4

Table 3 Optimization goals for problem C

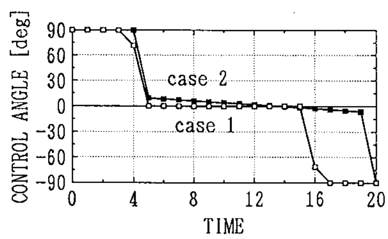
Objective	Priority for C1	Priority for C2
Vertical velocity ≤ 5.0	1	2
Final vertical velocity $= 0$	2	3
Final height $= 100$	3	1
Maximize final horizontal velocity	4	4



a) Flight trajectories x_1 vs x_2.



b) Time histories of velocity components x_3 and x_4 vs t.



c) Time histories of control angle β vs t.

Fig. 5 Solutions of problem C (ill-defined).

solved. A particle of mass m is guided by controlling the direction angle β of a thrust with magnitude ma on a planar inertial coordinate system x_1, x_2 (Fig. 2). This problem can be found in Refs. (1) and (23).

The state equations of motion are

$$x = (x_1, x_2, x_3, x_4)^T$$

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{Bmatrix} 0 \\ 0 \\ a\cos\beta \\ a\sin\beta \end{Bmatrix} \tag{15}$$

where x_3 and x_4 are the particle's velocities.

Let's consider the transfer of the particle to a specified height h with the zero vertical velocity in a fixed given time T so as to maximize the final horizontal velocity (problem A). The boundary conditions and performance index for the problem are

$$\psi_I = x(0) - \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = 0 \tag{16}$$

$$\psi_F = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x(T) - \begin{Bmatrix} h \\ 0 \end{Bmatrix} = 0 \tag{17}$$

$$\phi = -[0\ 0\ 1\ 0]x(T) \tag{18}$$

The time scale is divided into N elements ($\Delta t = T/N$) to convert the optimal control problem to a nonlinear programming problem. Therefore, the discrete control variables are represented as β_i [$i = 1, 2, \ldots, (N+1)$]. In the sequential GP formulation, the boundary conditions at the final time T should be dealt with as the goal functions since those conditions are not easily satisfied initially. Because the boundary conditions are more important than the minimization of the performance index, the goals associated with the boundary conditions and the goal for the performance index are classified into the first and the second propriety classes, respectively.

Figure 3 shows numerical examples where h, T, a, and N are designated as 100, 20, 1.123972, and 20, respectively. The initial variables were defined as a horizontal flight, i.e., the initial values of β_i were set to be 0. Figures 3a–3c show the calculated flight trajectory, the time histories of velocity components, and the control angle time history. In these figures, solid lines are the exact solutions, and "□" and "△" denote the calculated results. The relative error in the performance index between calculated results and analytical solutions was less than 0.5%. The iteration history of the flight trajectory is illustrated in Fig. 4, which shows that the goals in the upper class (the boundary conditions at the final time) are earlier satisfied and then the lower goal (the performance index) is improved subsequently.

Another example is a free-final-time problem (problem B) in which the final time (t_F) is minimized and the final horizontal velocity is specified as U. Boundary conditions at the final time and the performance index are

$$\psi_F = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x(T) - \begin{Bmatrix} h \\ U \\ 0 \end{Bmatrix} = 0 \tag{19}$$

$$\phi = t_F \tag{20}$$

and the independent variables are

$$\beta_1, \beta_2, \ldots, \beta_{(N+1)}, \quad t_F \tag{21}$$

In the numerical calculation, h, U, a, and N are set to be 100, 15, 1.695517, and 20, respectively. Boundary conditions at t_f and the minimization of t_F are classified into upper and lower priority classes, respectively.

The relative errors of the performance index, i.e., the maximum velocity $x_3(T)$ in problem A and the minimum time t_F in problem B, vs the number of elements are presented in Table 2. The accuracy of the solutions is improved by increasing the number of time elements. The relationship between the number of iterations and the number of elements is also summarized in Table 2. The criterion of convergence was defined as

$$\sum_i \frac{f_i^{k+1}(p) - f_i^k(p)}{1 + |f_i^k(p)|} < 1.0\mathrm{E} - 4 \tag{22}$$

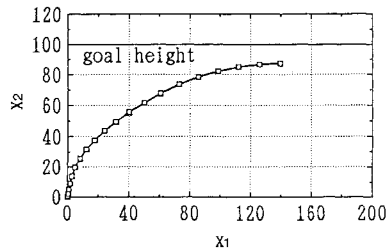
where the upper subscript means the iteration number. It is shown that the number of iterations is almost the same independently of the number of elements.

Table 4 Optimization goals for problem D

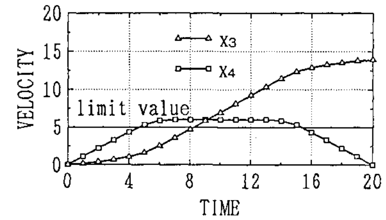
Priority	Objective
1	Final vertical velocity = 0
2	Fuzzy goals for final height, final horizontal velocity, and path constraints

Table 5 Membership function for problem D

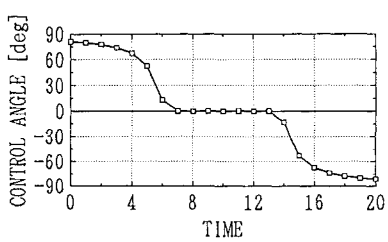
Goal	$\mu = 0$	$\mu = 1$
Final flight height	75.0	100.0
Final horizontal velocity	12.0	16.0
Maximum vertical velocity	7.0	5.0



a) Flight trajectory x_1 vs x_2.



b) Time histories of velocity components x_3 and x_4 vs t.



c) Time history of control angle β vs t.

Fig. 6 Solutions of problem D (fuzzy problem).

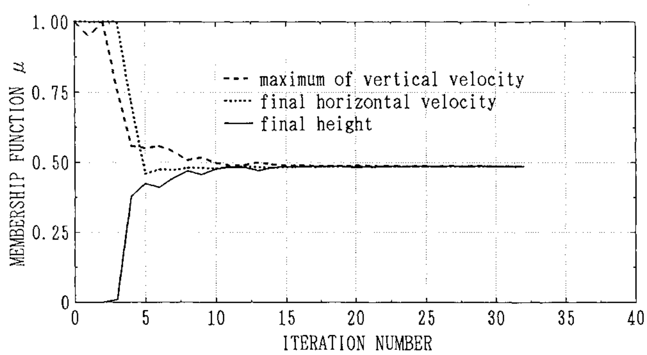


Fig. 7 Membership functions μ vs iteration number in problem D (fuzzy problem).

Ill-Defined Trajectory Optimization

Although the problems described in the preceding section are mathematically well-defined, real-world flight trajectory designs may be formulated as the ill-defined problems in which we can find no flight trajectory satisfying all boundary conditions or path constraints. In an ill-defined problem, there is no mathematical solution. However, the sequential GP formulation can solve the problem by trying to achieve the goals according to their priority levels. Here, we will examine an ill-defined flight trajectory problem that has tight path constraints.

In problem A, we assume that the vertical velocity is restricted under $U_{pc} = 5$ all through a flight path in order to consider tight path constraint situations. Table 3 illustrates prioritized design goals in two cases. In problem C1, the path constraints are most important, and subsequent goals are considered in the order of final vertical velocity, final height, and final horizontal velocity. In problem C2, the goals are classified in the order of final height, path constraints, final vertical velocity, and final horizontal velocity. Note that the classification of the priority levels has not changed during the iteration process.

Figure 5 presents the calculated flight trajectories for the two cases. Although the first and the second goals are satisfied in problem C1, final height does not reach the goal value of 100. In problem C2, on the other hand, only the final height (the first goal) is satisfied, and path constraints and final vertical velocity cannot obtain their goal values. In both cases, to satisfy the path constraints, control angles are maintained approximately at 0 deg in the middle of the flight path.

Trajectory Optimization with Fuzzy Decisions

We will again examine the ill-defined problem (problem C) with a limitation of the vertical velocity. The GP solutions for problem C1 (Fig. 5) indicated that the lower goals (final height and final horizontal velocity) were not easily satisfied due to the satisfaction of higher goals (the path constraints for the vertical velocity and the final vertical velocity). Table 4 classifies final height, final horizontal velocity, and path constraints in the same priority level.

The membership function used in Fig. 1 is determined by specifying two corner points, i.e., the best and worst cases. Table 5 shows these cases for three goals: final height, final horizontal velocity, and maximum vertical velocity (path constraint). The results are illustrated in Fig. 6. Although the path constraints are not satisfied in a strict sense, final height and final horizontal velocity have improved over those in problem C1 (Fig. 5). Figure 7 illustrates the iteration history of the values of membership functions of these three goals. Since an initial flight path is selected as a horizontal flight, which has no vertical velocity but a high horizontal velocity, membership functions associated with final horizontal velocity and maximum vertical velocity have a high degree of satisfaction in the early stage of iterations. However, in this stage, the membership function of final height has a low degree of satisfaction. Since the minimum value of all membership functions indicates the total satisfaction, the degree of satisfaction in the early stage is low. Whereas the values of membership functions of both final horizontal velocity and maximum vertical velocity decrease as iteration proceeds, the value of the membership function of final height increases. Consequently, the degree of satisfaction of the total system increases, and three membership functions converge to the same final values.

Conclusion

A sequential GP formulation was presented for the numerical optimization of an optimal flight trajectory. The method presented offers solutions for ascent trajectory problems in which initial design variables were far from optimal. Furthermore, the sequential GP method was successfully applied to ill-defined problems that have no feasible solutions satisfying design requirements such as boundary conditions and path constraints. Since the unsatisfied requirements are considered goals to be achieved in the GP formulation, ill-defined problems can be solved approximately by satisfying goals according to their priority levels. The fuzzy decision making method was applied to the sequential GP formulation presented for real-world flight trajectory designs in which the priority for each goal may be determined in a fuzzy manner. Numerical examples for ill-defined problems illustrate that an optimal flight trajectory can be calculated by the present method by prioritizing goals according to their significance and by defining membership functions for those goals at the same priority level. Finally, although a simple time integration method was successfully applied to the test examples, a more accurate method should be incorporated into the present optimization process to avoid an excessive increase in the number of design variables in more complicated problems.

Appendix: Modified Euler Method

First-order ordinary differential equations are presented as

$$\frac{dx}{dt} = \dot{x} = f(x, t) \tag{A1}$$

The formula for the Euler method is

$$x_{n+1} = x_n + h \cdot f(x_n, t_n) \tag{A2}$$

which advances a solution from t_n to $t_{n+1} = t_n + h$.

The modified Euler method is given as a more accurate integration formulation in the following equations:

$$\begin{aligned} \tilde{x}_{n+1} &= x_n + h \cdot f(x_n, t_n) \\ x_{n+1} &= x_n + h \cdot \frac{f(x_n, t_n) + f(\tilde{x}_{n+1}, t_{n+1})}{2} \end{aligned} \tag{A3}$$

References

[1]Bryson, A. E., Jr., and Ho, Y. C., *Applied Optimal Control,* Blaisdell, Waltham, MA, 1969.

[2]Hargraves, C. R., and Paris, S. W., "Direct Trajectory Optimization Using Nonlinear Programming and Collocation," *Journal of Guidance, Control, and Dynamics,* Vol. 10, No. 4, 1987, pp. 338–342.

[3]Betts, J. T., and Huffman, W. P., "Application of Sparse Nonlinear Programming to Trajectory Optimization," *Journal of Guidance, Control, and Dynamics,* Vol. 15, No. 1, 1992, pp. 198–206.

[4]Johnson, F. T., "Approximate Finite-Thrust Trajectory Optimization," *AIAA Journal,* Vol. 7, No. 6, 1969, pp. 993–997.

[5]Williamson, W. E., "Use of Polynomial Approximation to Calculate Suboptimal Controls," *AIAA Journal,* Vol. 9, No. 11, 1971, pp. 2271–2273.

[6]Rader, J. E., and Hull, D. G., "Computation of Optimal Aircraft Trajectories Using Parameter Optimization Methods," *Journal of Aircraft,* Vol. 12, No. 11, 1975, pp. 864–866.

[7]Hardtla, J. W., Piehler, M. J., and Bradt, J. E., "Guidance Requirements for Future Launch Vehicles," *Proceedings of the AIAA Guidance, Navigation, and Control Conference,* AIAA, New York, 1987.

[8]Charnes, A., and Cooper, W. W., *Management Models and Industrial Applications of Linear Programming,* Vol. 1, Wiley, New York, 1961.

[9]Ijiri, Y., *Management Goals and Accounting for Control,* Rand McNally, Chicago, 1965.

[10]Lee, S. M., *Goal Programming for Decision Analysis,* Auerbach Publishers, Philadelphia, PA, 1972, pp. 15–35.

[11]Ignizio, J. P., *Linear Programming in Single- & Multiple-Objective Systems,* Prentice-Hall, Englewood Cliffs, NJ, 1982.

[12]Suzuki, S., and Matsuda, S., "Structure/Control Design Synthesis of Active Flutter Suppression System by Goal Programming," *Journal of Guidance, Control, and Dynamics,* Vol. 14, No. 6, 1991, pp. 1260–1266.

[13]Suzuki, S., and Yonezawa, S., "Simultaneous Structure/Control Design Optimization of a Wing Structure with a Gust Load Alleviation System," *Journal of Aircraft,* Vol. 30, No. 2, 1993, pp. 268–274.

[14]Bellman, R. E., and Zadeh, L. A., "Decision Making in a Fuzzy Environment," *Management Science,* Vol. 17, No. 4, 1970, pp. 141–164.

[15]Zadeh, L. A., "Fuzzy Sets," *Information and Control,* Vol. 8, 1965, pp. 338–353.

[16]Zimmermann, H. J., "Decision and Optimization of Fuzzy Systems," *International Journal of General Systems,* Vol. 2, No. 4, 1976, pp. 209–215.

[17]Zimmermann, H. J., "Fuzzy Programming and Linear Programming with Several Objective Functions," *Fuzzy Sets and Systems,*

Vol. 1, No. 170, 1978, pp. 45–55.

[18]Wang, G. Y., and Wang, W. Q., "Fuzzy Optimal Design of Structures," *Engineering Optimization,* Vol. 8, No. 4, 1985, pp. 291-300.

[19]Rao, S. S., "Description and Optimum Design of Fuzzy Mechanical Systems," *Journal of Mechanism, Transmission, and Automation in Design,* Vol. 109, No. 1, 1987, pp. 126-132.

[20]Dhingra, A. K., Rao, S. S., and Miura, H., "Multiobjective Decision Making in a Fuzzy Environment with Applications to Helicopter Design," *AIAA Journal,* Vol. 28, No. 4, 1990, pp. 703–710.

[21]Miele, A., Prichard, R. E., and Damoulakis, J. N., "Sequential Gradient-Restoration Algorithm for Optimal Control Problems," *Journal of Optimization Theory and Applications,* Vol. 5, 1970, pp. 235-282.

[22]Vanderplaats, G. N., *Numerical Optimization Techniques for Engineering Design,* McGraw-Hill, New York, 1984, pp. 155–157.

[23]Hodges, D. H., and Bless, R. R., "Weak Hamiltonian Finite Element Method for Optimal Control Problems," *Journal of Guidance, Control, and Dynamics,* Vol. 14, No. 1, 1991, pp. 148–156.